Scale effects related to rating curve of cylindrically crested PKWs

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Content

1. Introduction

1. Piano Key Weirs (PKWs) spill extraordinary high discharges under small heads
2. Physical models are often used to support design process
3. Small prototype heads result in diminutive model heads
4. Diminutive model heads are prone to scale effects

Key question: up to what limit can we trust physical models?

Van Phong Vietnam (HoTaKhan)

Loombah Dam for Benalla (Australia)

1. Introduction

Scale effects in physical modelling

Prototype vs. physical model

Geometrical scale factor 1:40

For a reliable rating curve derived by scaled physical models: minimal heads are to be respected

Saint marc dam PKW (France)

Saint marc dam PKW, model at LCH

2. Scale effects

- Geometrical length scaling with factor λ
- Hydraulic scaling with dynamic similitude to compare forces acting on water

Isaac Newton: the sum of all forces acting on an object (water volume) is equal to its mass times the related acceleration (inertia).

Main forces acting on water

- Weight (gravity)
- Pressure field
- Viscous forces
- Surface tension

Main goal: Forces *F* acting in a model should be the same (relative to inertia *I*) as in the prototype (dynamic similitude).

$$\frac{F_m}{I_m} = \frac{F_p}{I_p}$$

2. Scale effects

Gravity is the dominant driving force in free-surface flows: Froude similarity

$$\frac{I}{F} = \frac{ma}{mg} = \frac{l^3\delta}{l^3\delta}\frac{l}{t^2}\frac{1}{g} = \frac{l^2}{t^2}\frac{l^2}{l^3}\frac{1}{g} = \frac{V^2}{lg} = \mathsf{F}^2$$

Froude similitude and geometrical scaling:

$$V_p = \frac{\sqrt{gh_p}}{\sqrt{gh_M}} = \sqrt{\lambda}V_M \qquad t_p = \sqrt{\lambda}t_m$$

Similitude of turbulent forces

$$\mathrm{R_P} = \frac{\left(\sqrt{\lambda}V_M\right)\left(\lambda l_M\right)}{\nu} = \lambda^{3/2}R_M$$

Similitude of surface forces

$$\mathrm{W_P} = \frac{\rho V^2 h}{\sigma} = \lambda^2 W_M$$

A true dynamic similarity over falls requires identical Froude, Reynolds and Weber numbers in prototype and model. This is physically impossible

2. Scale effects

Minimum heads necessary to avoid relevant scale effects related to rating curve

Reference		*Application rate [m]*	*Shape*	*R [m]*
Bollrich and Aigner	(2000)	H_m=0.04÷0.06	cylindrical weir	
Rehbock	(1909)	H_m=0.03÷0.05	sharp-crested	
Dillmann	(1933)	H_m=0.05÷0.07	sharp-crested	
Kirschmer	(1928)	H_m=0.07	cylindrical weir	R=0.046
Sarginson	(1972)	H_m=0.05	sharp-crested	
Sarginson	(1972)	H_m=0.05	cylindrical weir	R=0.03
Breitschneider	(1978)	H_m=0.02	standard ogee	
Breitschneiderx	(1978)	H_m=0.06	sharp-crested	
Hager and Schwalt	(1994)	H_m=0.05	broad-crested	
Ettema	(2000)	H_m=0.025	rating curve	
Ettema	(2000)	H_m=0.06	nappe shape	

Typically

H<0.03÷0.05 m

3. Approaches

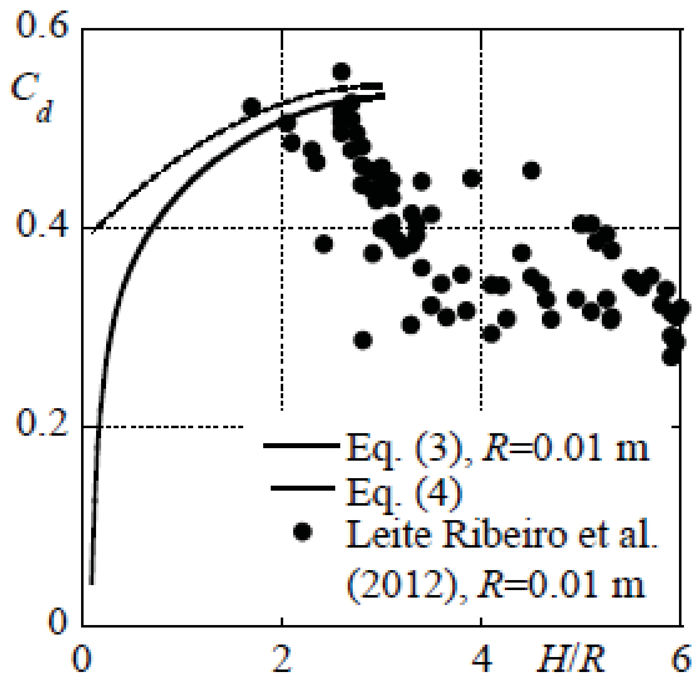
Contraction on edges and corners are ignored
(straight cylindrical weir = PKW with diminutive head)

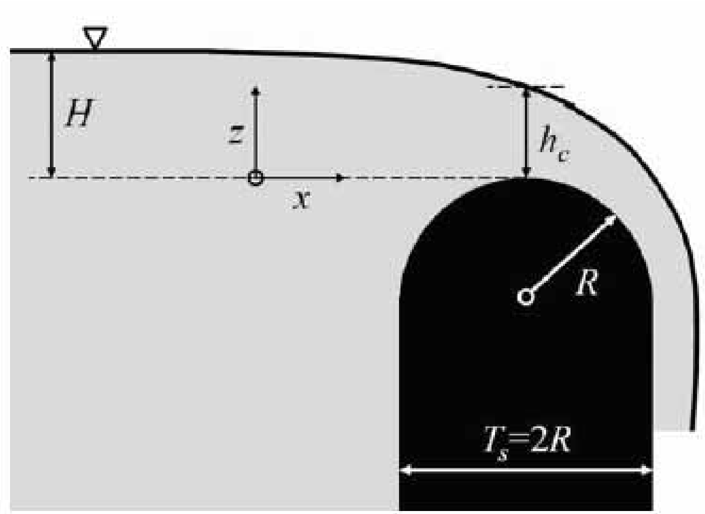
- ➢ Numerical simulation

Model dimensions: T_s=0.005÷0.02 m
Prototype dimensions: T_s=0.20÷0.35 m

- Navier-Stockes equation
- Turbulence model
- Two equation k-Ɛ model, for viscous effect
- Static contact angle, for surface tension effect
- Isothermal simulation at 20° C
- Surface roughness, wall shear stress calculations
- Fine rectangular mesh, 0.5 mm high cells along *z*

- ➢ Theoretical approach

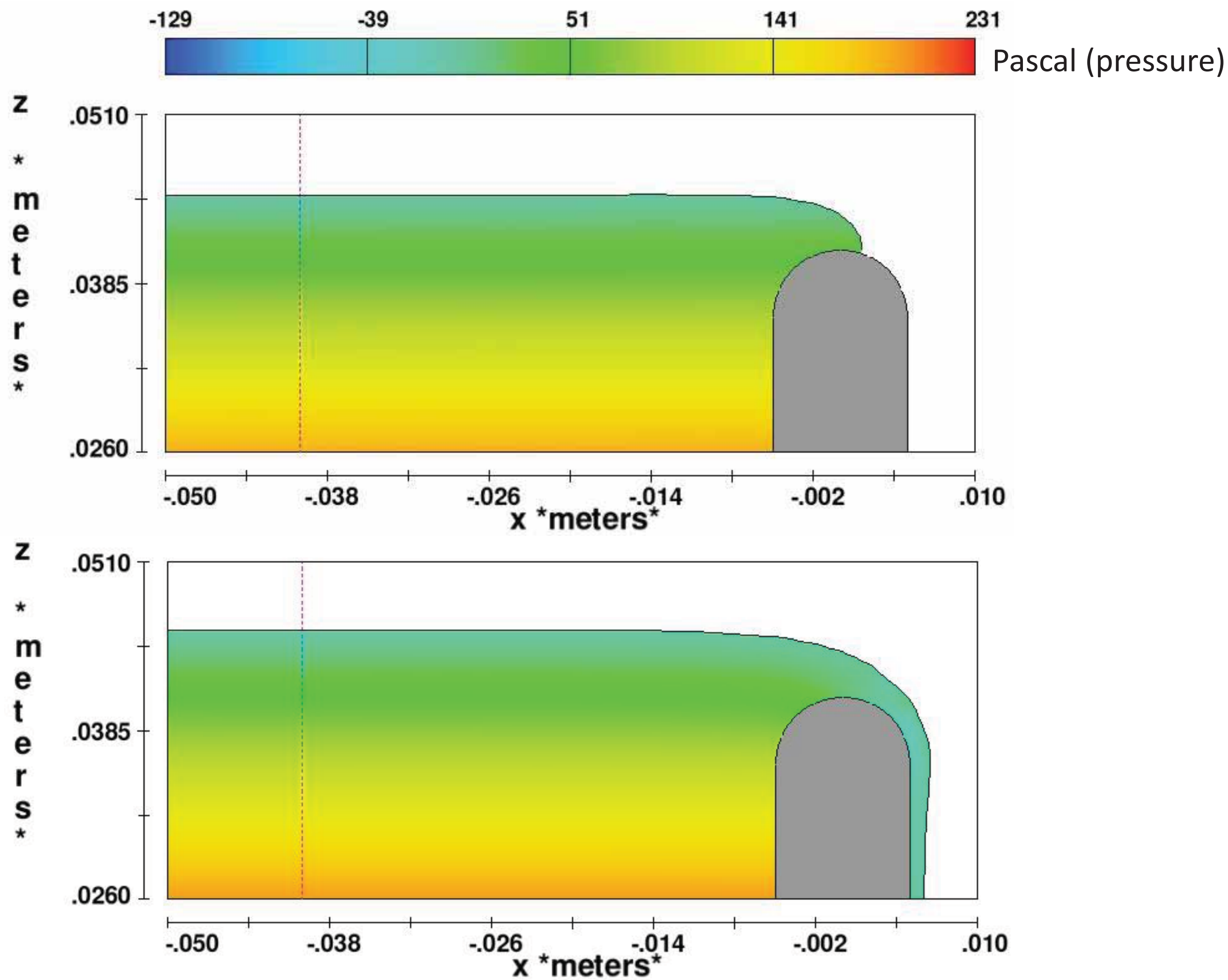




3.1 Numerical simulation

Results

- Limitation 1 → onset of over-flow, limited by surface tension
- Limitation 2 → flow retarded by surface tension and viscosity



3.2 Theoretical approach

Theoretical discharge coefficient
for cylindrical weir crest:

$$q = \frac{Q}{L} \qquad C_d = \frac{q}{\sqrt{2gH^3}}$$

according to Matthew (1991) and Castro-Orgaz (2012):

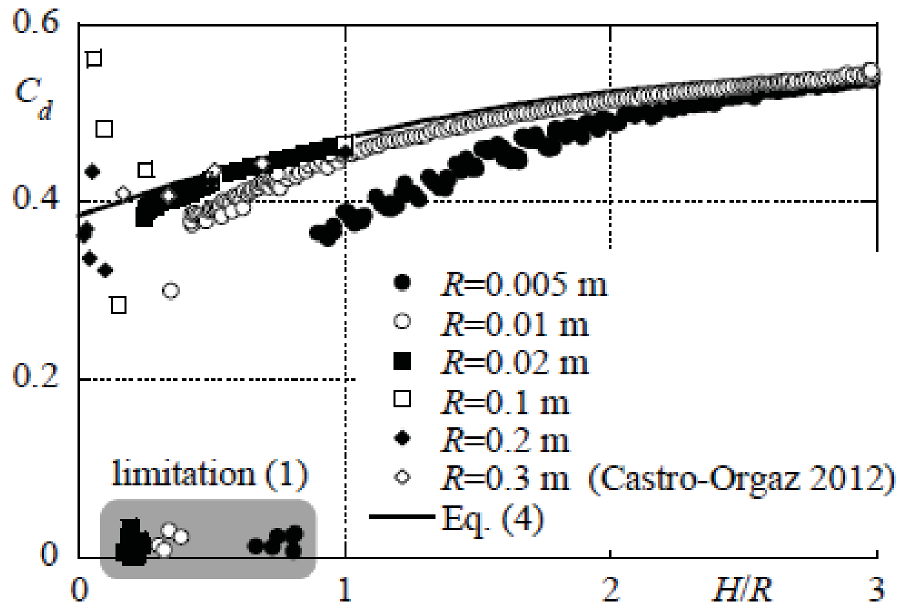
$$C_d = 0.385\left[1 + 0.272\left(\frac{H}{R}\right) - 0.833\left(\frac{\sigma}{\rho g R^2}\right)\left(\frac{R}{H}\right) - 1.05\left(\frac{3}{g}\right)^{\frac{1}{4}}\sqrt{\nu}R^{\frac{-3}{4}}\left(\frac{R}{H}\right) - 0.045\left(\frac{H}{R}\right)^2\right]$$

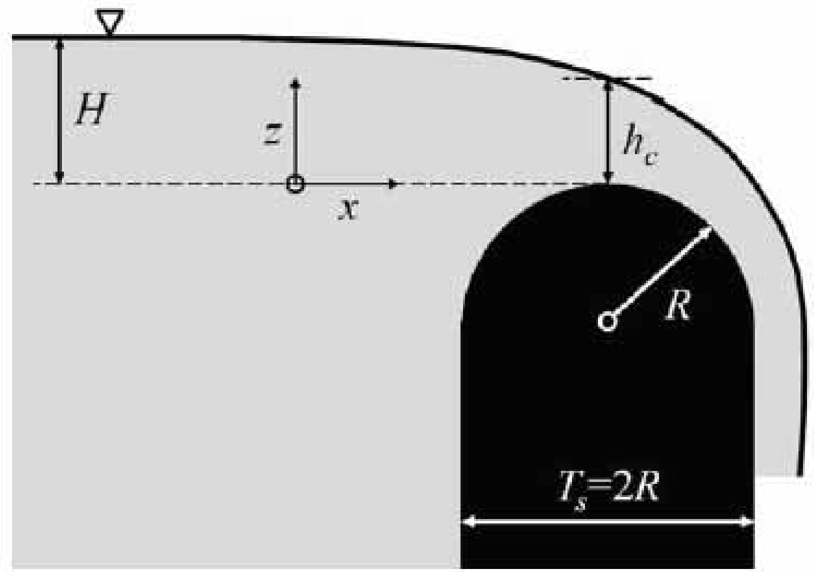
For potential flow ($\nu=\sigma=0$):

$$C_d = 0.385\left[1 + 0.272\left(\frac{H}{R}\right) - 0.045\left(\frac{H}{R}\right)^2\right]$$

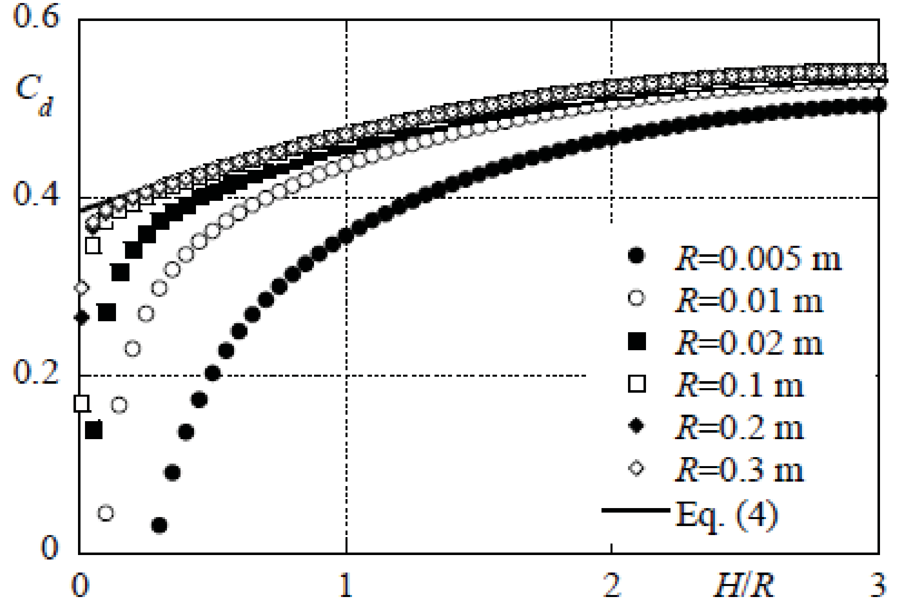
4. Comparison of approaches

Numerical model





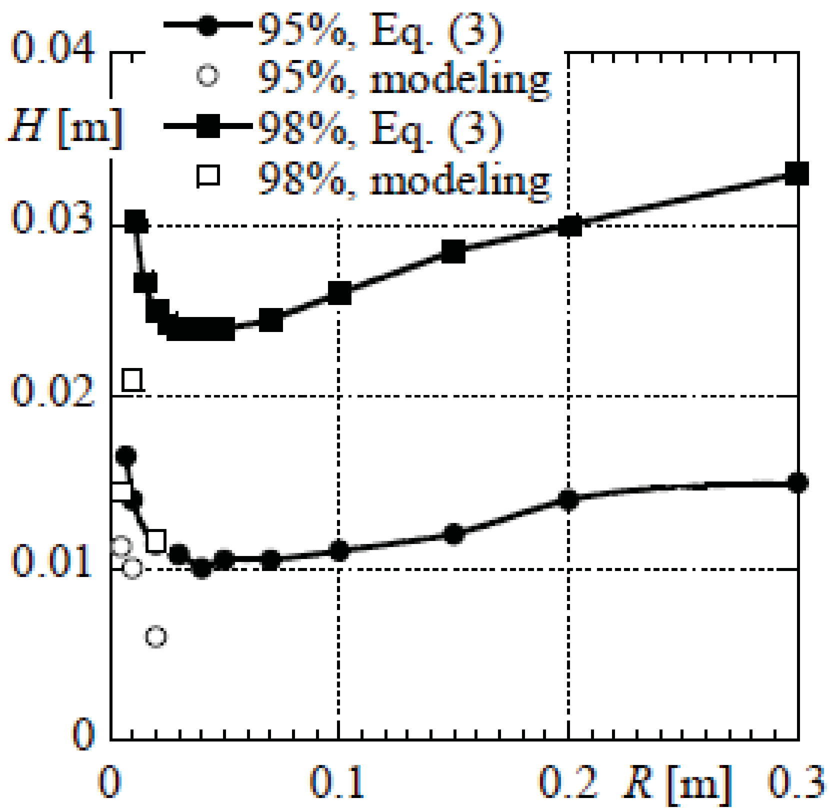
Theory



4. Comparison of approaches

Conclusions

- Limitation (1): H=0.004÷0.005 m
- Limitation (2): >2% error for H<0.03 m
- Small "model" R generate C_d below those of "prototype" → rating curve is incorrectly (but on safe side)
- Scale effects related to rating curve for large relative heads disappear



5. Conclusion

- Rating curve on PKWs is independent of folded structure if $H/R<2$ and discharge coefficient related to developed crest length
- Limitation (1): onset of flow, H=0.004÷0.005 m
- Limitation (2): surface tension and viscosity retard flow up to potential flow, which is achieved at around H=0.03 m
- Physical models underestimate the discharge for H<0.03 m

Thank you for the attention.

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